NO ARBITRAGE SABR BY USING DEEP NEURAL NETWORKS AND GPUS

eigenView Inc.

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ABOUT EIGENVIEW INC.

- Company

  eigenView Inc.

  Established in 2007

  - Representative

    Kumagai, Masatoshi

    Resume - a few financial software companies, Fuji bank, Keio univ.

  - Business area

    Computer system development specialized market/credit risk management, ALM and derivative pricing for financial institutions.

ABSTRACT

- Next-generation function approximation using deep neural network (DNN) and GPUs will be shown in this presentation
  - Based on Universal Approximation Theorem for neural networks
- As an example, new **no arbitrage SABR approximation** method to interpolate and extrapolate volatilities is prototyped
  - Foreign exchange option is a target transaction type (if no negative rate, could also be applied to interest rate option)
- Training data for DNN is generated by Monte Carlo (MC) simulation with GPUs
  - These data imitate FX market volatilities
  - GPU can rapidly generate a large number of MC paths for a short time.
- DNN produce calibrated SABR parameters and no arbitrage SABR approximation function through training
  - Numerical result will be shown later in this document
- Then, user can easily calibrate SABR parameters to the market and quickly get volatilities they need by using this technology

HOW TO DRAW SMILE CURVE USING NEW METHOD (1/15)

- Outline
  - Universal Approximation Theorem says that neural networks can approximate any function.
  - Brief procedure on how to operate DNN
    1. Training data generation
       - *GPU generates a large amount of training data which obey SABR model using MC simulation*[1]
       - A lot of combinations of SABR variables and parameters compose computational elements for DNN training
       - Range of variables and parameters should cover real market volatility smile
    2. Learning

       DNN learns relation between feature and label data[2] by analyzing generated data above and produces SABR approximation formula through DNN regression

       In the case of SABR model approximation,

       - feature data : α_0,β,ρ,ν, *strike, tenor, forward rate*
       - label data : σ_{black}

    3. Inferring
       - Infer and check values with other data generated by GPU and market data

SABR model

$$dF_t = \alpha_t F_t^{\beta} dW_1$$
$$d\alpha_t = \nu \alpha_t dW_2$$
$$\rho = dW_1 dW_2$$

Approximation formula f()

f(α_0,β,ρ,ν, *forward rate, strike, tenor*) = σ_{black}

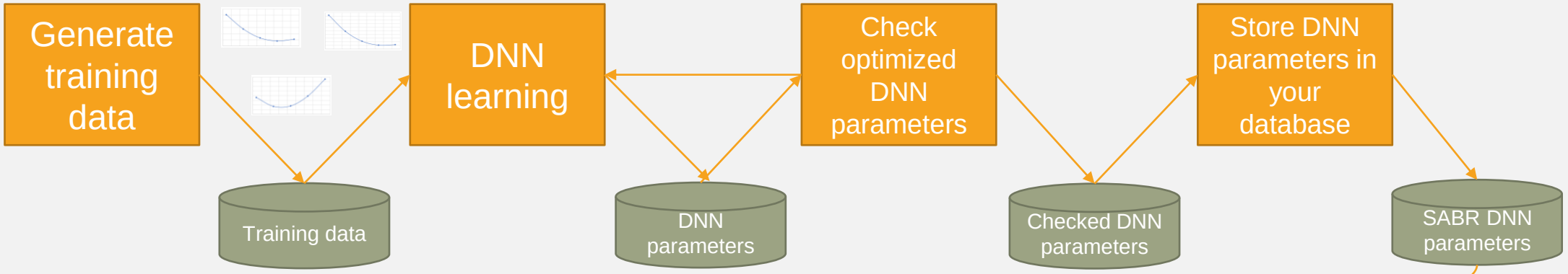
If no problem, let's golive!

1 If you have another method which is faster and more accurate to compute a volatility obeying SABR model than MC by GPU, you should use such a method. Parallel FDM can be a candidate?

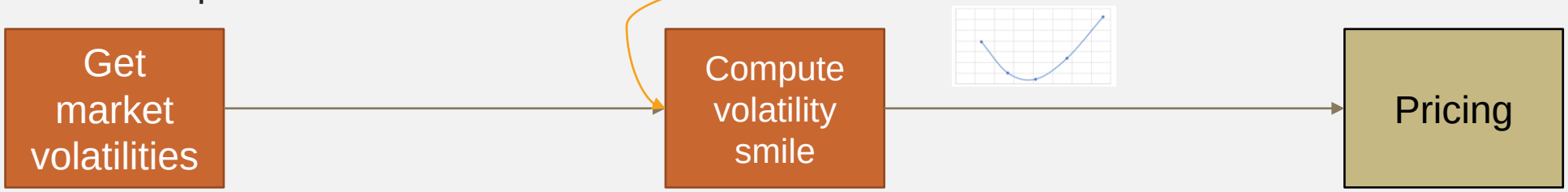
2 Both feature and label data compose training data.

HOW TO DRAW SMILE CURVE USING NEW METHOD (2/15)

- Upfront work



- Valuation in practice



HOW TO DRAW SMILE CURVE USING NEW METHOD (3/15)

- Interpolation / extrapolation for FX implied volatility

1. Calibration

There are two calibrating options.

I. SABR function approximation (DNN Approx.)

SABR parameters are calibrated to market volatilities[3] by using both this approximation which is produced from DNN and a non-linear optimization tool, such as Levenberg–Marquardt algorithm.

II. Direct calibration (DNN Calib.)

DNN can directly infer SABR parameters from market volatilities[3] by learning relation between volatilities and SABR parameters through training. The parameters are used as the initial value of the calibration by DNN Approx above.

2. Interpolation/extrapolation

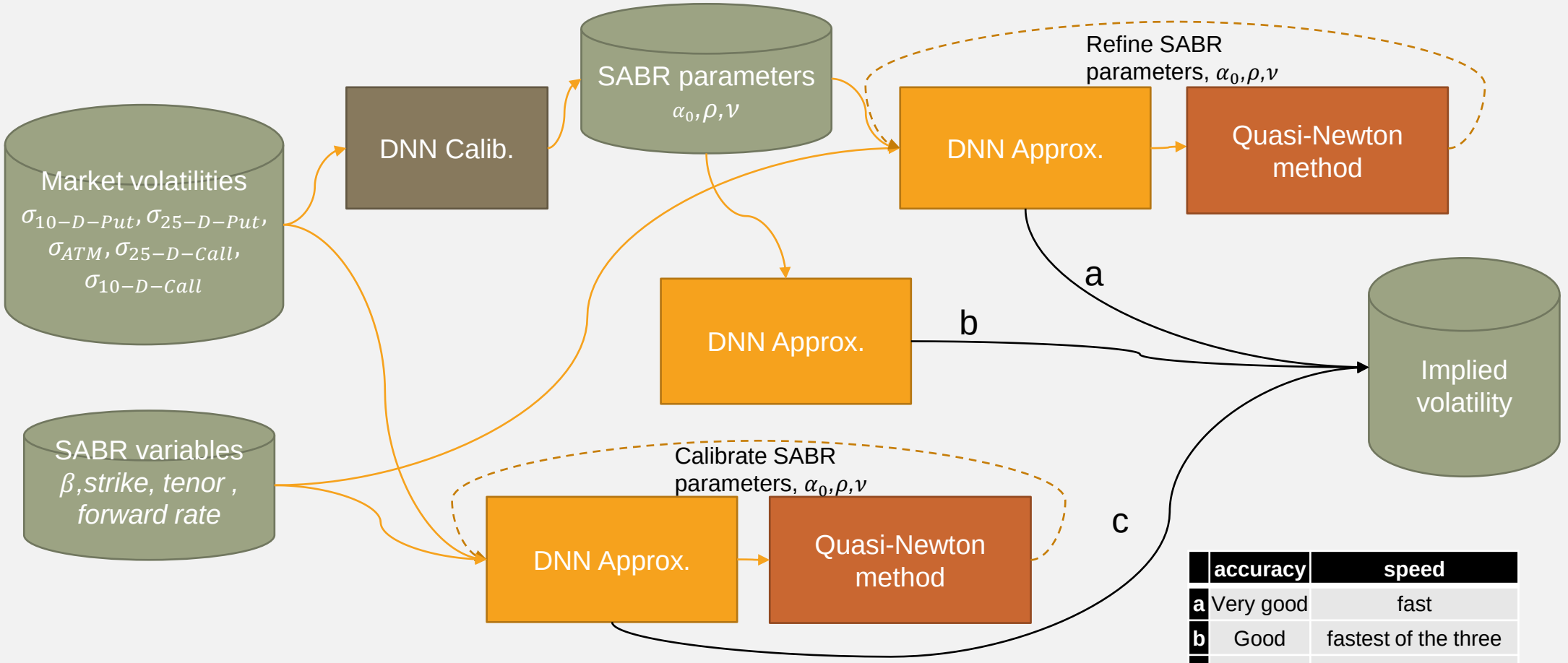
I. The parameters and the strike are substituted into DNN Approx. tool.

II. Implied volatility at the strike is calculated.

3 They are called as 10%-delta-put, 25%-del-put, ATM, 25%-delta-call, and 10%-delta-call.

HOW TO DRAW SMILE CURVE USING NEW METHOD (4/15)

- Schematic view in terms of the relation between DNN Approx. and DNN Calib.



	accuracy	speed
a	Very good	fast
b	Good	fastest of the three
c	Good?	slow

HOW TO DRAW SMILE CURVE USING NEW METHOD (5/15)

SABR model

$$dF_t = \alpha_t F_t^{\beta} dW_1$$
$$d\alpha_t = \nu \alpha_t dW_2$$
$$\rho = dW_1 dW_2$$

- Generating training (feature and label) data (1/2)
  - Using GPU
    - By using GPU, Black implied volatility (IV) obeying SABR model is rapidly computed through Monte Carlo simulation
    - To cover possible strike range and as training data, Greeks-delta should be calculated simultaneously
  - Training data contents

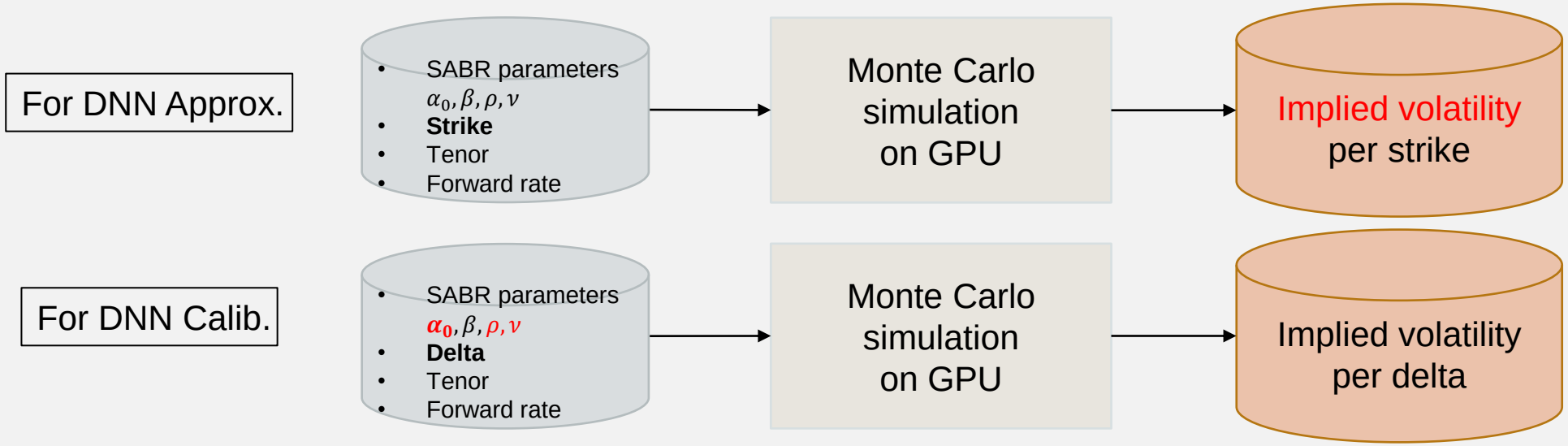
Produced tool	Feature data	Label data	Remarks
DNN Approx.	• SABR parameters α_0, β(fixed)[4], ρ, ν • Strike • Tenor • Forward rate	Implied volatility	➢ Tenors are matched to market convention. ➢ Strike and IV are generated by GPU
DNN Calib.	• Five imitated market volatilities $\sigma_{10-D-Put}, \sigma_{25-D-Put}, \sigma_{ATM}, \sigma_{25-D-Call}, \sigma_{10-D-Call}$ • SABR parameter β(fixed)[4] • Tenor • Forward rate	SABR parameters α_0, ρ, ν	➢ Imitated market volatility data are generated by GPU. **The data are also useful for calibration test for DNN Approx.** ➢ Tenors are matched to market convention.

4 $\beta$is fixed according to market convention

HOW TO DRAW SMILE CURVE USING NEW METHOD (6/15)

- Generating training (feature and label) data (2/2)
  - Feature data
    - Random distance - by random numbers, easy to increase data densely later

      or
    - Equal distance - easy to generate, but difficult to increase data densely later
  - Label data
    - *Every implied volatility σ_{black} is generated to coincide to every combination of feature data*



HOW TO DRAW SMILE CURVE USING NEW METHOD (7/15)

- Monte Carlo simulation for SABR model

$$dF_t = \alpha_t F_t^{\beta} dW_1$$
$$d\alpha_t = \nu \alpha_t dW_2$$
$$\rho = dW_1 dW_2$$

Time discretization … Euler-Maruyama method

$$F_{i+1} = F_i + \alpha_i F_i^{\beta} \sqrt{\Delta T} w_1$$

$$\alpha_{i+1} = \alpha_i + \nu \alpha_i \sqrt{\Delta T} \left(\rho w_1 + \sqrt{1-\rho^2} w_2 \right) \leftarrow \boldsymbol{fast, but\ not\ so\ accurate}$$

or

$$\begin{pmatrix} \alpha_{i+1} = \alpha_i \cdot exp\left[-\frac{1}{2}\nu^2 \Delta T + \nu\sqrt{\Delta T}\left(\rho w_1 + \sqrt{1-\rho^2} w_2 \right) \right] \\ or \\ log(\alpha_{i+1}) = log(\alpha_i) + \left[-\frac{1}{2}\nu^2 \Delta T + \nu\sqrt{\Delta T}\left(\rho w_1 + \sqrt{1-\rho^2} w_2 \right) \right] \end{pmatrix} \leftarrow \boldsymbol{accurate, but\ very\ slow}$$

$$ForwardValue_K = \frac{1}{M} \sum_j^M \left[max(F_{j,T} - K, 0) \right]$$

- Computing deltas

search $\sigma_{Black,delta} and\ K_{delta},\ delta = \{10\_D_{put}, 25\_D_{put}, ATM, 25\_D_{call}, 10\_D_{call}\}$

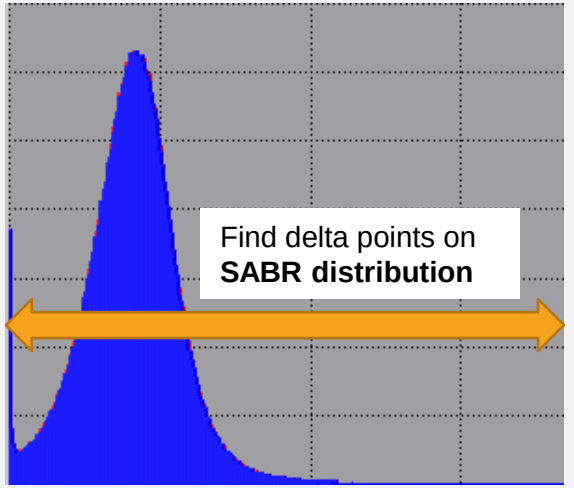
that satisfy

$$\boldsymbol{\Delta_{F;pips}} = \boldsymbol{N}\left(\frac{\boldsymbol{log}\left(\frac{F_0}{K_{delta}}\right) + \frac{\mathbf{1}}{\mathbf{2}}\sigma^2_{Black,delta} T}{\sigma_{Black,delta}\sqrt{\boldsymbol{T}}} \right) = \{0.9, 0.75, 0.5, 0.25, 0.1\}$$

subject to

$$FV_{Black,delta}(F_0, K_{delta}, T, \sigma_{Black.delta}) = FV_{SABR,delta}(F_0, K_{delta}, T, \alpha_0, \beta, \rho, \nu)$$

on Monte Carlo paths.



HOW TO DRAW SMILE CURVE USING NEW METHOD (8/15)

- Technique for reduction of generating trading data
  - Training data of DNN Calib. can be shared for currency pairs which are the same delta type and have the same β.
    - FX option quote types in market convention
      - Premium unadjusted … EUR/USD, GBP/USD, AUD/USD, and so on
      - Premium adjusted … most other currency pairs
    - Four types of market convention delta.

	delta	
Tenor	≤ 1 year	> 1 year
Premium unadjusted	$\phi e^{-r^f T} N(\phi d_1)$	$\phi N(\phi d_1)$
Premium adjusted	$\phi e^{-r^d T} \frac{K}{S_0} N(\phi d_2)$	$\phi \frac{K}{F_0} N(\phi d_2)$

	Formula
d_1	$= \frac{\boldsymbol{log}\left(\frac{F_0}{K}\right) + \frac{\mathbf{1}}{\mathbf{2}}\sigma^2 T}{\sigma\sqrt{\boldsymbol{T}}}$
d_2	$= d_1 - \sigma\sqrt{\boldsymbol{T}}$
S_0	$= F_0 e^{(r^f - r^d)T}$

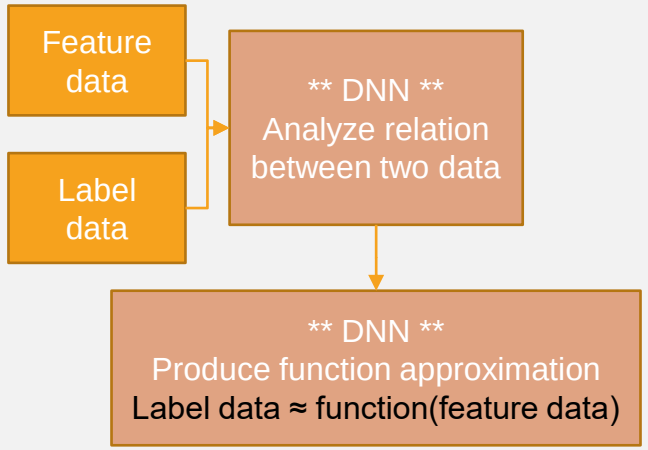
  - Transformation : $F_0^{training} = \frac{F_0}{F_0} = 1 \;\&\; K^{training} = \frac{K}{F_0}, S_0^{training} = \frac{S_0}{S_0} = 1 \;\&\; K^{training} = \frac{K}{S_0}, \alpha_0^{training} = \frac{\alpha_0}{F_0^{1-\beta}}$
  - By means of these transformations, only four type of training data can cover all currency pairs and both tenor ≤ 1 year and tenor > 1 year.

HOW TO DRAW SMILE CURVE USING NEW METHOD (9/15)

- DNN Learning setting (1/2)
  - DNN regression analysis

SABR parameters	DNN Approx.	DNN Calib.
α_0	Feature	Label
β	Fixed	Fixed
ρ	Feature	Label
ν	Feature	Label
Forward rate	Not used (due to all value is 1)	Not used
Strike	Feature	Not used
Tenor	Feature	Feature
Implied volatility	Label	Feature
Interest rate	Not used	Feature (≤ 1 year)
Remarks		Multiple outputs

"Label" data are equal to independent variable of regression analysis.



  - Both data volumes should be at lest 10 thousands? Not sure.
  - Certain ratio of training data should be allocated to validation data to check overfitting.
  - Certain ratio of training data should be allocated to test data to check validity of DNN model

HOW TO DRAW SMILE CURVE USING NEW METHOD (10/15)

- DNN Learning setting (2/2)
  - Machine learning setting

Machine learning parameters	Detail	Remarks
Loss function	Mean squared error	• There are a lot of manners to populate DNN parameters.
Activation function	ReLU, linear, tanh, …	
Optimization	RMSProp, ADAM, …	
Number of layers	3 -	
Number of nodes	32 -	
Batch size	64 -	
Learning rate	0.001 -	
… … … … … … … … … …	… … … … … …	

HOW TO DRAW SMILE CURVE USING NEW METHOD (11/15)

- Accuracy and precision about Monte Carlo simulation

  To show accuracy and precision of MC, some example charts in terms of comparison with other SBAR computing methods below will be shown on the following page.

  - Comparison with two tools
    - Hagan approximation
    - Finite Difference Method (FDM)

FDM setting:

using Quantlib FdSabrVanillaEngine

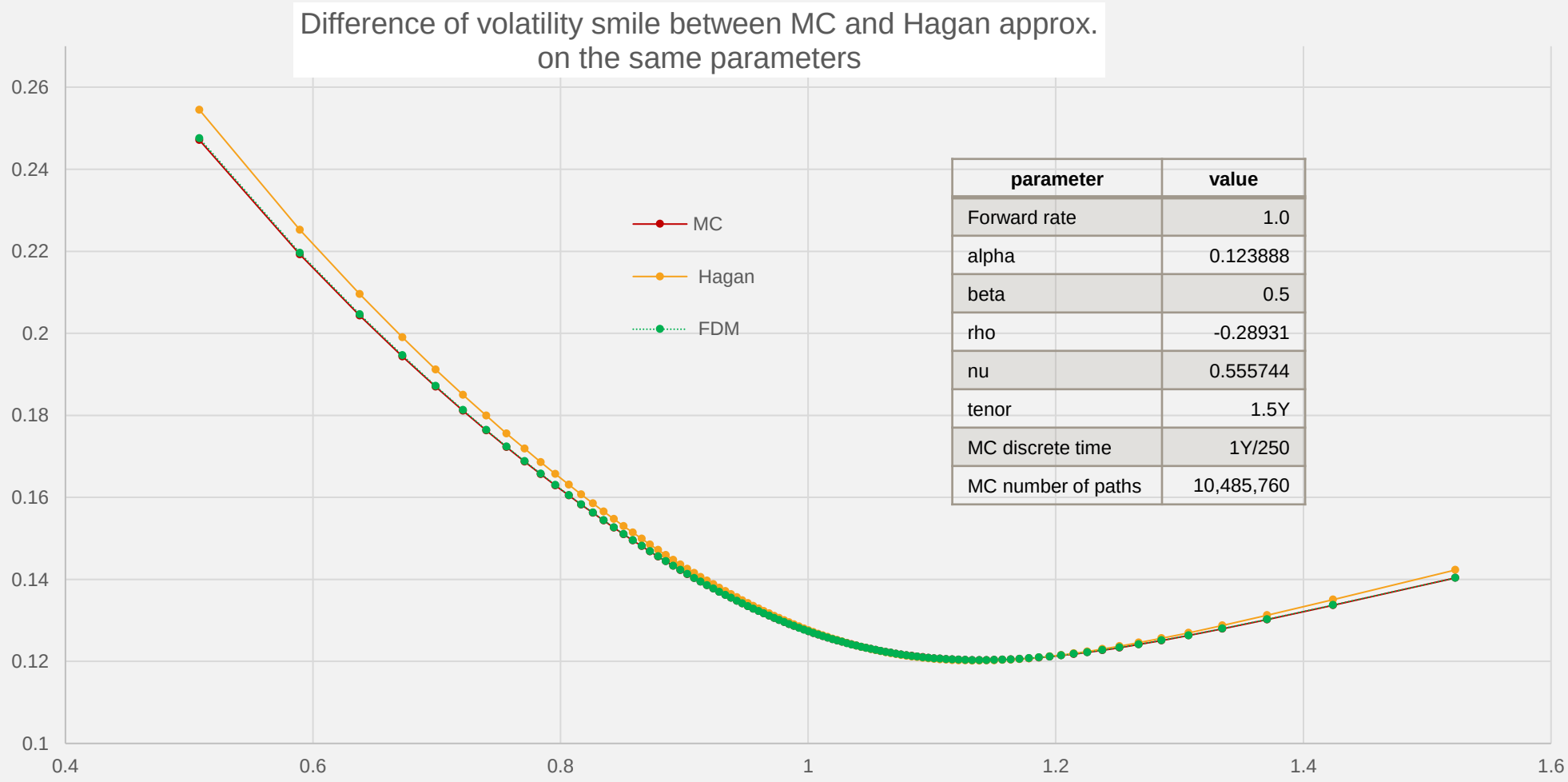
Parameter	Value	Remarks
tGrid	100	Number of time discretization
fGrid	400	Number of underlying price discretization
xGrid	100	Number of volatility of volatility discretization

The other parameters are not changed.

The Hagan approximation is not changed.

HOW TO DRAW SMILE CURVE USING NEW METHOD (12/15)

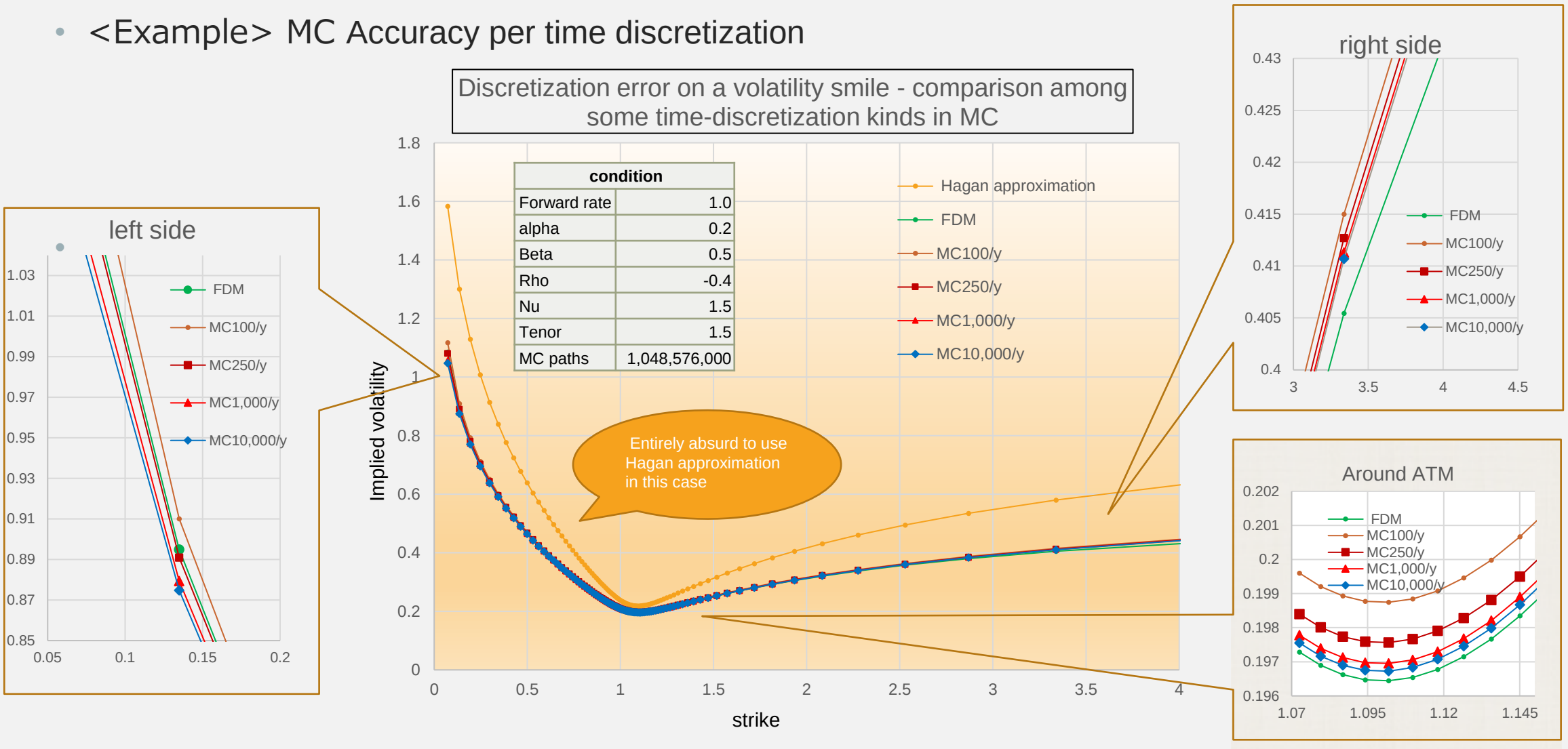
- Smile example



parameter	value
Forward rate	1.0
alpha	0.123888
beta	0.5
rho	-0.28931
nu	0.555744
tenor	1.5Y
MC discrete time	1Y/250
MC number of paths	10,485,760

HOW TO DRAW SMILE CURVE USING NEW METHOD (13/15)

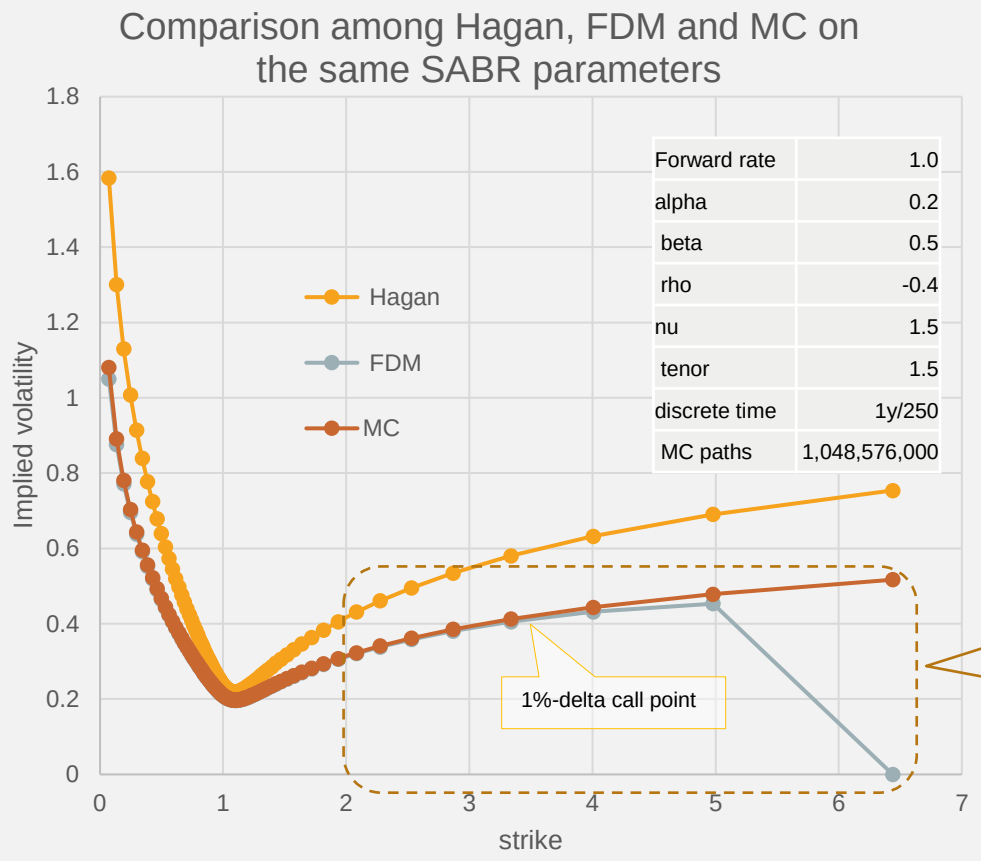
- <Example> MC Accuracy per time discretization

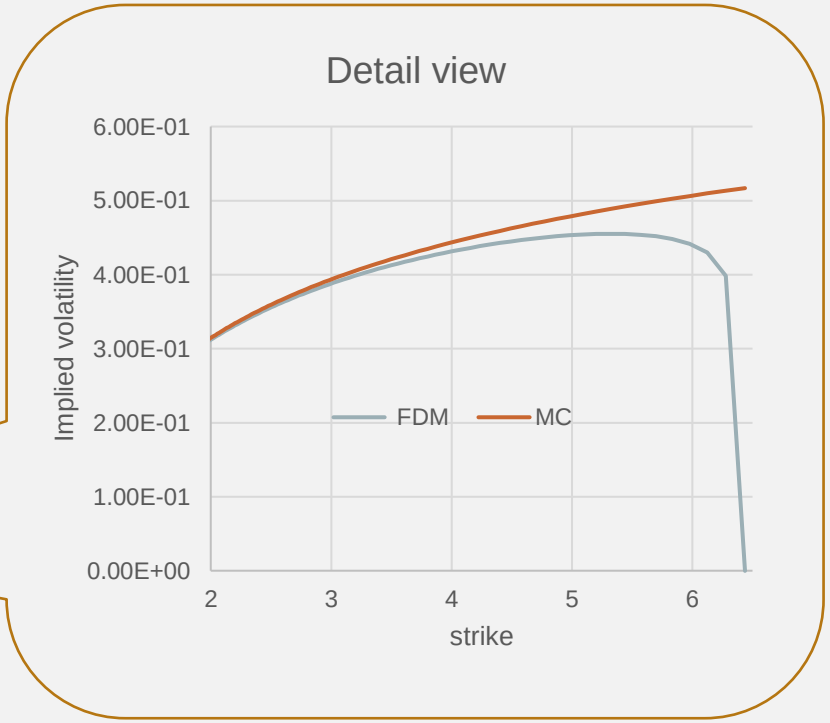


HOW TO DRAW SMILE CURVE USING NEW METHOD (14/15)

- <Example> FDM error

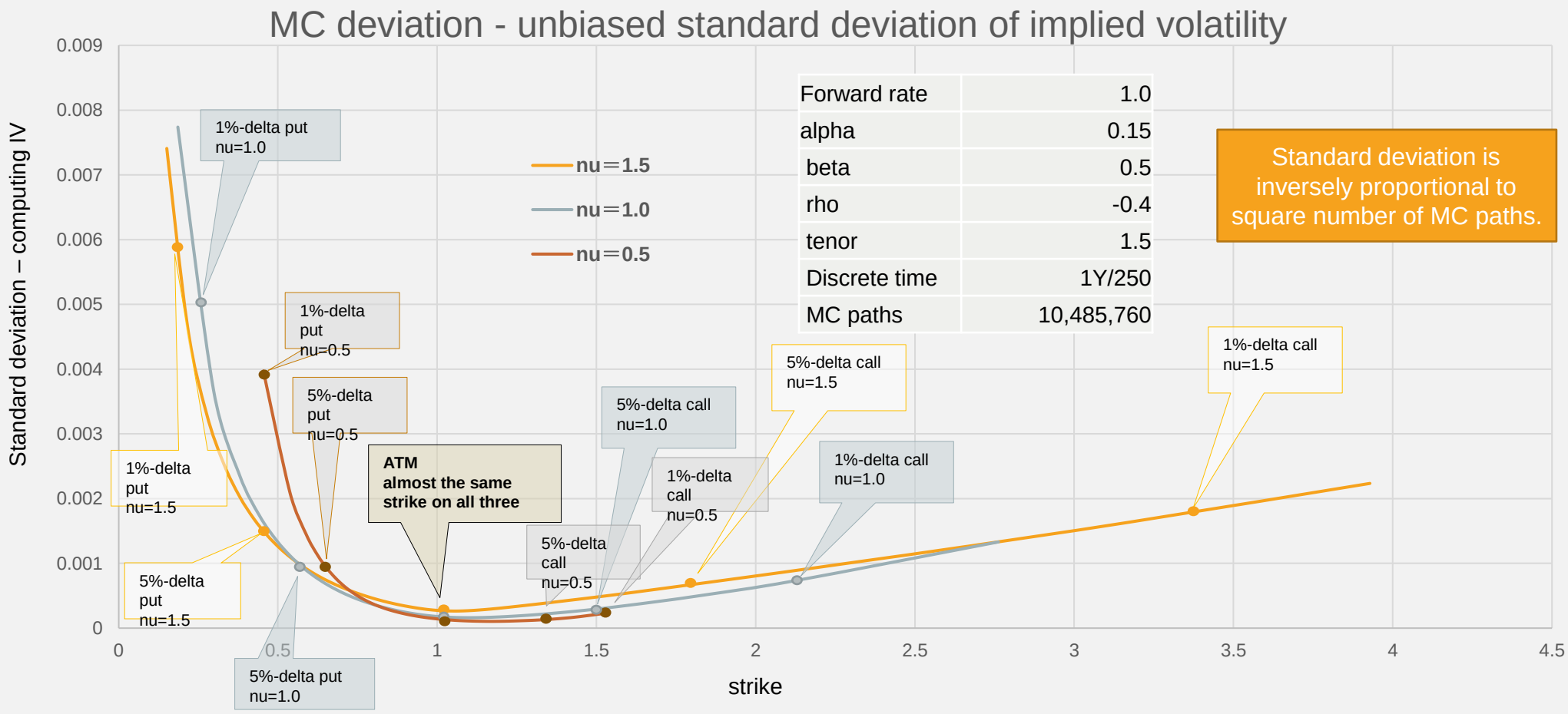
Upper and lower boundary for variable is indispensable in FDM.
Then on the near boundary, computed value is often wrong.





HOW TO DRAW SMILE CURVE USING NEW METHOD (15/15)

- <Example> MC Precision



PROTOTYPE TEST (1/19)

- Setting for generating training data (1/2)

	Prototype setting	Remarks
PGM language	CUDA v.11.7	
Tenor	1.5 year	For 1 or less year, interest rate is needed to compute delta. It is troublesome for this prototype. This value means **375/250, business day base.**[5]
Currency pair	EUR/USD	
Delta	7 quoted deltas	Adding 5%-delta put and 5%-delta call as loqer/upper limits of strikes
Strike range	5%-delta put ≤ strike ≤ 5%-delta call	Extremely small or big strike value is omitted. Most users mightn't prefer extrapolation in far wings.
α	0.05 – 0.3	Using random number generator
β	Fixing to 0.5	
ρ	-0.9 – 0.9	Using random number generator
ν	0.05 – 1.5	Using random number generator
MC Discrete tool	Euler-Maruyama	
MC Discrete time	1y/250	
MC Number of paths	10 million	
Data precision	Float32 (single precision)	Significant digits of market volatility is covered enough by float32.

5 In practice, it is needed to generate data per business days or calendar days, or "tenor" should be as feature data instead of fixed value.

PROTOTYPE TEST (2/19)

- Setting for generating training data (2/2)

  - Data image

All data in the same line are generated at once for a few seconds by using GPU.

Input data example

fwdrate	tenor	alpha	beta	rho	volvol
1	1.5	0.065315357	0.5	-0.35811	0.525657
1	1.5	0.242011988	0.5	0.863899	1.260464
1	1.5	0.165297476	0.5	0.180486	0.989391
1	1.5	0.156737171	0.5	-0.03611	0.054857
…	…	…	…	…	…
1	1.5	0.284696963	0.5	0.183503	0.858997
1	1.5	0.243779974	0.5	-0.71787	1.42026

output data example

σ5dp	σ10dp	σ25dp	σATM	σ25dc	σ10dc	σ5dc	K5dp	K10dp	K25dp	KATM	K25dc	K10dc	K5dc	Strike	Volatility
0.095434	0.085922	0.074367	0.066575	0.063093	0.063264	0.064822	0.830754	0.878689	0.944325	1.00333	1.056652	1.107715	1.143092	1.063823	0.062926
0.25222	0.213786	0.212114	0.289299	0.467208	0.754917	0.97634	0.631036	0.739874	0.868074	1.064782	1.73267	5.0145	14.61181	13.2204	0.957622
0.285801	0.23774	0.193168	0.184423	0.213578	0.272548	0.323987	0.597805	0.718378	0.876704	1.025837	1.234473	1.621734	2.078002	1.836106	0.298777
0.16994	0.166625	0.161354	0.156006	0.151172	0.147252	0.14508	0.72565	0.786072	0.892468	1.018421	1.1526	1.280673	1.360776	1.219458	0.149054
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
0.447552	0.382823	0.322718	0.309517	0.341956	0.404549	0.455178	0.471716	0.612042	0.828217	1.074495	1.447999	2.133395	2.922271	2.669083	0.440883
0.605392	0.466604	0.308858	0.214163	0.180222	0.190915	0.210938	0.388796	0.566046	0.832272	1.034998	1.189148	1.386789	1.581401	1.293205	0.18271

  - Output data description

Output	Description	Usage
σ5dp - σ5dc	Seven volatilities, $\sigma_{5\%d\ put}, \sigma_{10\%d\ put}, \sigma_{25\%d\ put}, \sigma_{ATM}, \sigma_{25\%d\ call}, \sigma_{10\%d\ call}, and\ \sigma_{5\%d\ call}$	• As training data for DNN Approx. and DNN-calibration except both 5% put and call • For calibration test for DNN Approx.
K5dp - K5dc	Seven strikes, $K_{5\%d\ put}, K_{10\%d\ put}, K_{25\%d\ put}, K_{ATM}, K_{25\%d\ call}, K_{10\%d\ call}, and\ K_{5\%d\ call}$	For checking smile curve to draw
Strike	Chosen any value from $K_{5\%d\ put}$ to $K_{5\%d\ call}$	As feature data for DNN Approx.
Volatility	Black implied volatility coincided with the strike above.	As label data for DNN Approx.

PROTOTYPE TEST (3/19)

- Setting for DNN learning

	DNN Approx.	DNN Calib.	Remarks
Machine learning library	TensorFlow 2.8.0	TensorFlow 2.8.0	
Loss function	MSE	MSE	
Activation function	ReLU	ReLU, linear,…	
Optimization	ADAM	ADAM	
Epochs	5,000	5,000	
Bach size	8,192	4096	Using tf.data.Dataset
Number of training data	720,000	144,000	
Number of validation data	80,000	16,000	
Number of test data	200,000	40,000	
Number of layers	6	9	Details are confidential matter
Number of nodes	Max 256	Max 512	Details are confidential matter
Number of inputs	4	5	Tenor, β, and forward rate are fixed.
Number of outputs	1 (implied volatility)	3 (α_0, ρ, ν)	

PROTOTYPE TEST (4/19)

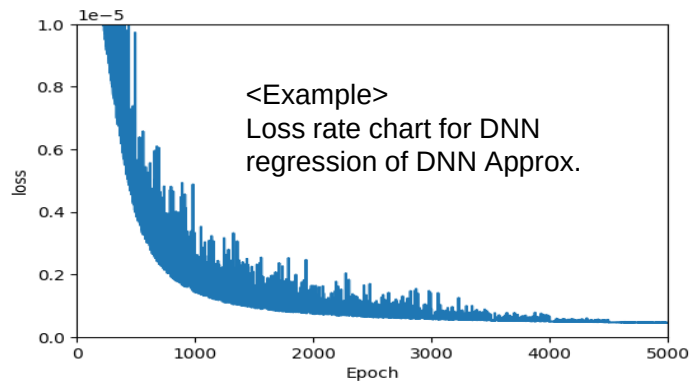
- Executing TensorFlow

  Every parameter should be being modified until some important indices of DNN show good values.

  It needs to accumulate experiences.

<tensorFlow log example - learning DNN regression of calibration>

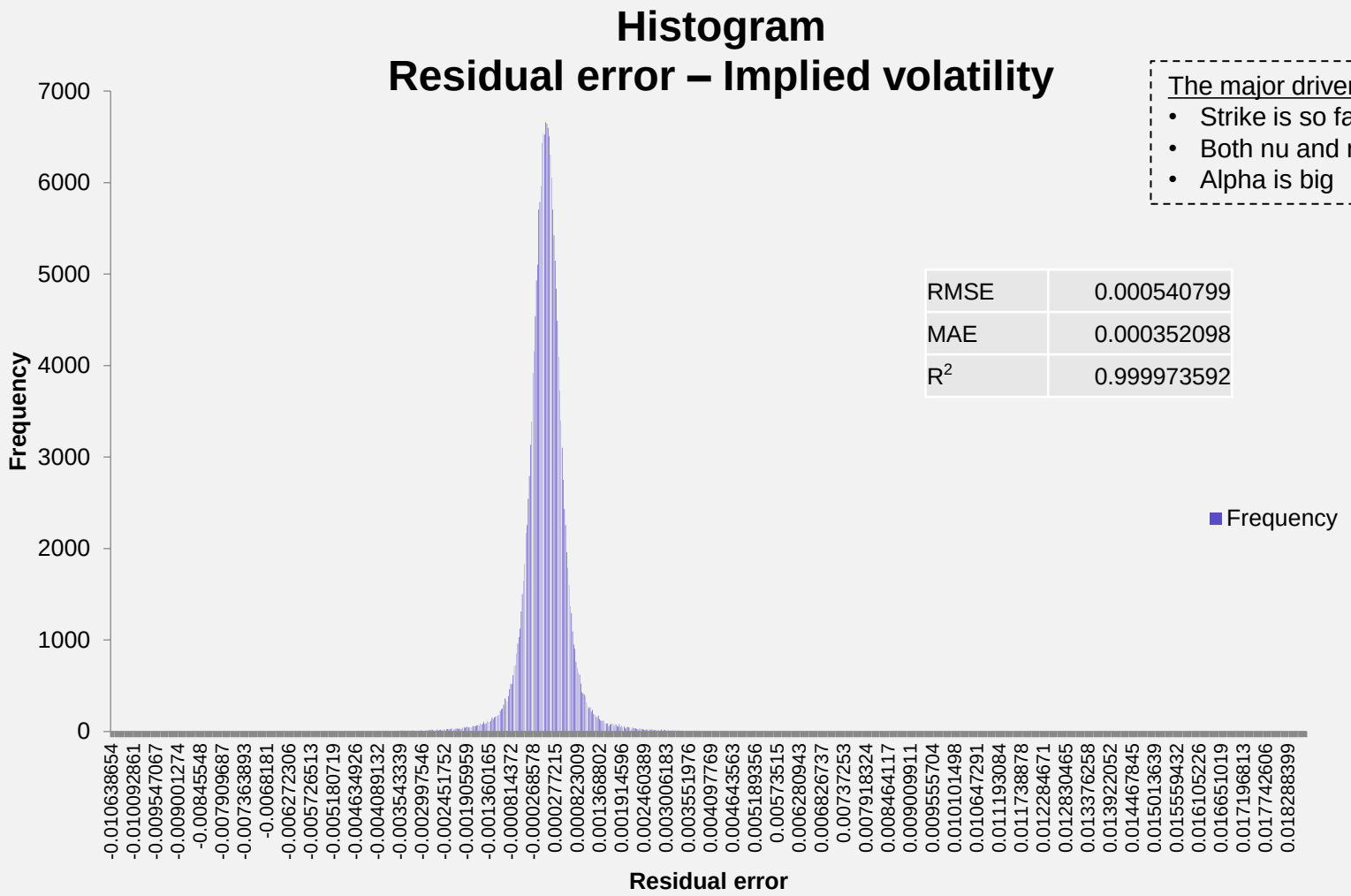
```
Epoch 1/5000
INFO:tensorflow:batch_all_reduce: 30 all-reduces with algorithm = nccl, num_packs = 1
INFO:tensorflow:Reduce to /job:localhost/replica:0/task:0/device:CPU:0 then broadcast to ('/job:localhost/replica:0/task:0/device:CPU:0',).
INFO:tensorflow:Reduce to /job:localhost/replica:0/task:0/device:CPU:0 then broadcast to ('/job:localhost/replica:0/task:0/device:CPU:0',).
INFO:tensorflow:Reduce to /job:localhost/replica:0/task:0/device:CPU:0 then broadcast to ('/job:localhost/replica:0/task:0/device:CPU:0',).
INFO:tensorflow:batch_all_reduce: 30 all-reduces with algorithm = nccl, num_packs = 1
6/6 [==============================] - 9s 429ms/step - loss: 0.8472 - alpha_loss: 1.7812 - rho_loss: 0.2859 - volvol_loss: 0.3798 - alpha_mae: 1.3328 - alpha_mse: 1.7812 - rho_mae:
0.4624 - rho_mse: 0.2859 - volvol_mae: 0.5365 - volvol_mse: 0.3798 - val_loss: 0.8261 - val_alpha_loss: 1.7473 - val_rho_loss: 0.2529 - val_volvol_loss: 0.3741 - val_alpha_mae: 1.3200 -
val_alpha_mse: 1.7473 - val_rho_mae: 0.4353 - val_rho_mse: 0.2529 - val_volvol_mae: 0.5321 - val_volvol_mse: 0.3741
Epoch 2/5000
6/6 [==============================] - 1s 194ms/step - loss: 0.8108 - alpha_loss: 1.7230 - rho_loss: 0.2328 - volvol_loss: 0.3660 - alpha_mae: 1.3108 - alpha_mse: 1.7230 - rho_mae:
0.4166 - rho_mse: 0.2328 - volvol_mae: 0.5236 - volvol_mse: 0.3660 - val_loss: 0.7932 - val_alpha_loss: 1.6897 - val_rho_loss: 0.2075 - val_volvol_loss: 0.3601 - val_alpha_mae: 1.2981 -
val_alpha_mse: 1.6897 - val_rho_mae: 0.3918 - val_rho_mse: 0.2075 - val_volvol_mae: 0.5188 - val_volvol_mse: 0.3601
Epoch 3/5000
```



<Example>
Loss rate chart for DNN regression of DNN Approx.

PROTOTYPE TEST (5/19)

- DNN result of DNN approx.

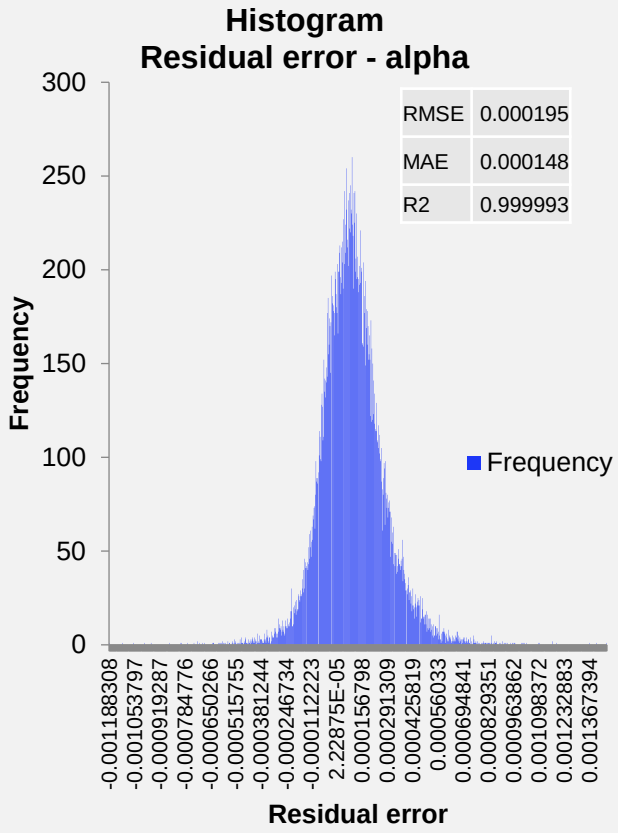


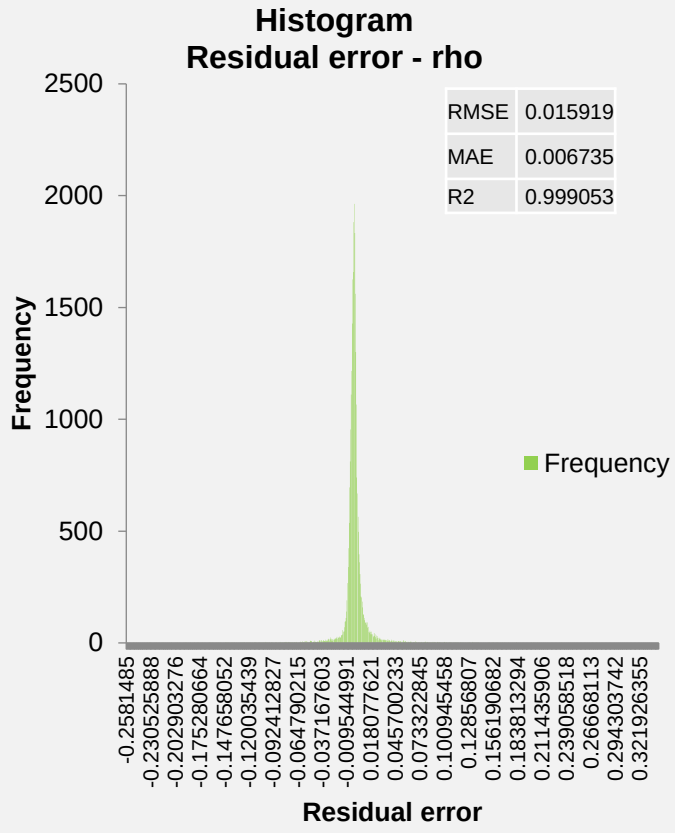
The major driver of error
- Strike is so far from ATM
- Both nu and rho are big
- Alpha is big

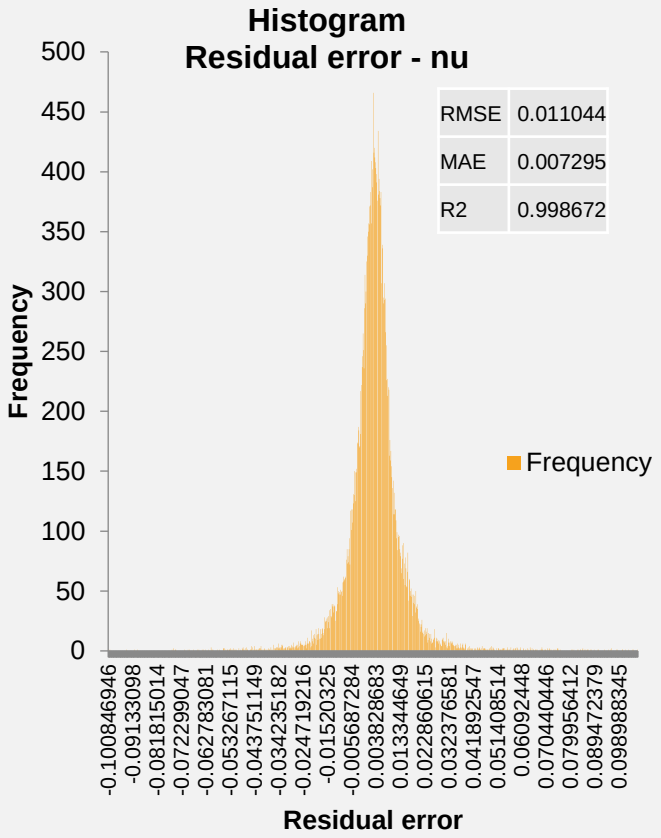
RMSE	0.000540799
MAE	0.000352098
R^2	0.999973592

PROTOTYPE TEST (6/19)

- DNN result of DNN Calib.







The causes of error are the same as DNN approx. in the last page.

PROTOTYPE TEST (7/19)

- Numerical result – drawing smile by DNN Approx. (1/2)
- Data example
  - setting

	Case 1	Case 2	Case 3	Case 4
Forward rate	1.0	1.0	1.0	1.0
tenor	1.5	1.5	1.5	1.5
α	0.1	0.1	0.2	0.2
β	0.5	0.5	0.5	0.5
ρ	-0.5	0	-0.5	0.5
ν	0.5	0.5	1.5	1.5

  - Numerical result --- these are also shown graphically in the next page.

Case 1			
strike	FDM	DNN Approx	difference
0.744988	0.15273	0.153583	0.000853345
0.817327	0.135825	0.135751	-7.39E-05
0.860714	0.12647	0.12625	-0.000220031
0.892419	0.120033	0.119972	-6.19E-05
0.917862	0.115139	0.115228	8.99E-05
0.93947	0.111193	0.111505	0.000311844
0.958557	0.10789	0.10805	0.000160024
0.97593	0.105047	0.105023	-2.45E-05
0.992136	0.102552	0.102595	4.30E-05
1.007584	0.100328	0.100494	0.000165835
1.022611	0.098322	0.098631	0.000308983
1.037527	0.096496	0.096737	0.000240341
1.052655	0.094825	0.094995	0.000169508
1.068381	0.093291	0.093606	0.000314832
1.085222	0.091886	0.092194	0.000307918
1.103983	0.090616	0.090717	0.000100926
1.126118	0.089512	0.089784	0.000271902
1.154866	0.08868	0.08888	0.000199206
1.200745	0.088558	0.088498	-6.05E-05

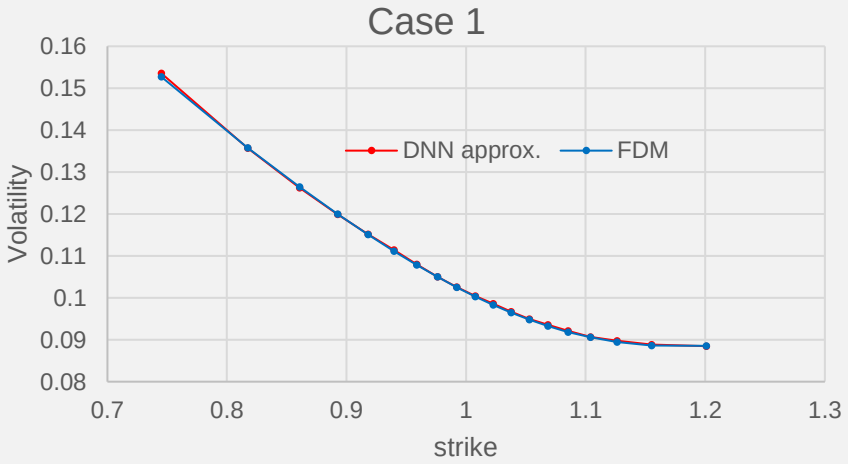
Case 2			
strike	FDM	DNN Approx	difference
0.781724	0.127177	0.127106	-7.17E-05
0.839064	0.117714	0.117754	4.07E-05
0.874329	0.112842	0.112663	-0.00017932
0.900848	0.10973	0.109248	-0.00048204
0.922784	0.107551	0.107548	-2.73E-06
0.942018	0.105956	0.106147	0.00019082
0.959581	0.104769	0.104866	9.72E-05
0.97613	0.103891	0.104042	0.00015128
0.992136	0.103264	0.103422	0.00015758
1.007983	0.102855	0.10281	-4.55E-05
1.024023	0.102648	0.1025	-0.0001476
1.040625	0.102641	0.10245	-0.00019132
1.058229	0.102846	0.10278	-6.53E-05
1.077415	0.103294	0.103285	-9.09E-06
1.099035	0.104043	0.104378	0.0003352
1.124495	0.105203	0.105413	0.00021067
1.156453	0.106989	0.106679	-0.00031014
1.201044	0.10991	0.109848	-6.23E-05
1.278944	0.115646	0.11543	-0.00021568

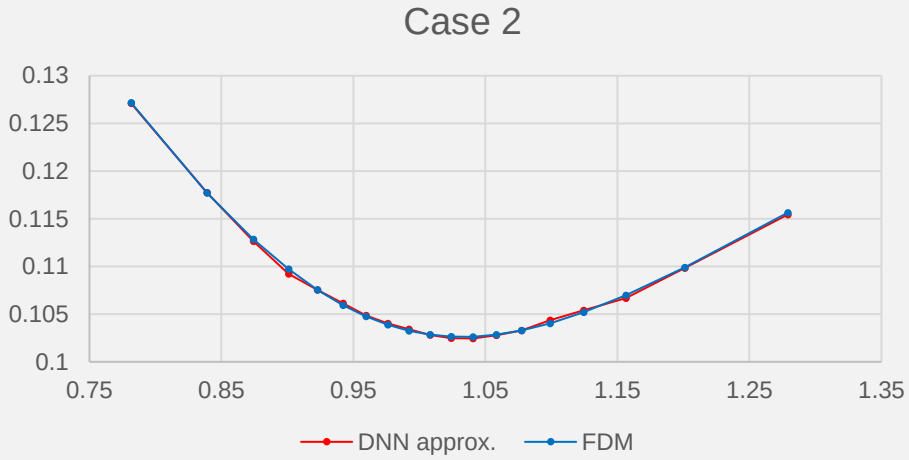
Case 3			
strike	FDM	DNN Approx	difference
0.30069	0.639747	0.621811	-0.01793599
0.501761	0.466258	0.470156	0.00389779
0.642868	0.374972	0.376667	0.00169501
0.745378	0.317491	0.319065	0.00157374
0.822366	0.278126	0.279773	0.00164709
0.882052	0.249899	0.250678	0.00077982
0.929849	0.229145	0.229794	0.00064841
0.96952	0.213738	0.21421	0.00047167
1.003868	0.202357	0.203814	0.00145756
1.035196	0.194167	0.195308	0.00114141
1.065665	0.188678	0.189899	0.0012209
1.097674	0.185705	0.18688	0.00117552
1.134424	0.185409	0.186307	0.00089839
1.180995	0.188433	0.189262	0.00082877
1.246842	0.196195	0.197011	0.00081591
1.352948	0.211606	0.213127	0.00152114
1.557525	0.241021	0.24142	0.00039996
2.091997	0.300515	0.30145	0.00093496

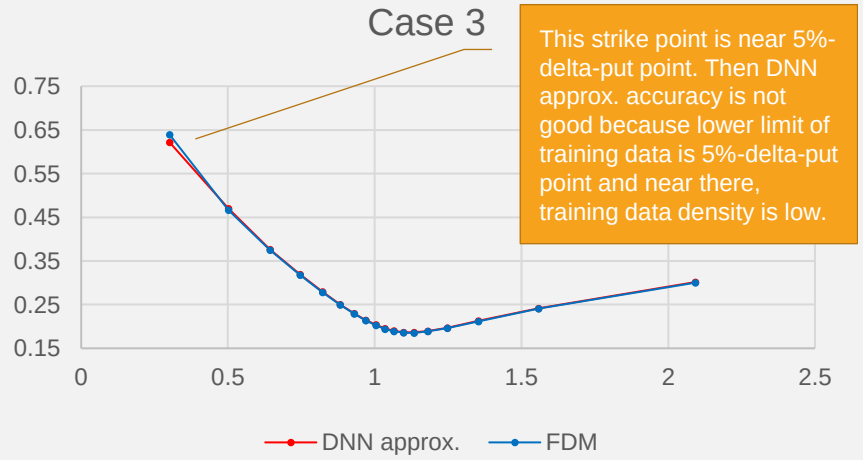
Case 4			
strike	FDM	DNN Approx	difference
0.429082	0.434533	0.432679	-0.00185373
0.631139	0.314932	0.321948	0.00701639
0.741561	0.264958	0.271092	0.00613421
0.811164	0.239924	0.244146	0.00422134
0.86051	0.227082	0.232413	0.00533143
0.899608	0.221371	0.227595	0.00622414
0.934303	0.220558	0.226351	0.00579239
0.968813	0.223799	0.229645	0.00584607
1.007065	0.231095	0.236724	0.0056289
1.053788	0.243097	0.248412	0.00531507
1.116006	0.261097	0.264906	0.00380808
1.205923	0.28721	0.29144	0.00423074
1.34777	0.324801	0.331852	0.00705174
1.59667	0.379176	0.388411	0.009235
2.09807	0.457561	0.472812	0.01525077
3.286737	0.559591	0.595245	0.03565383

PROTOTYPE TEST (8/19)

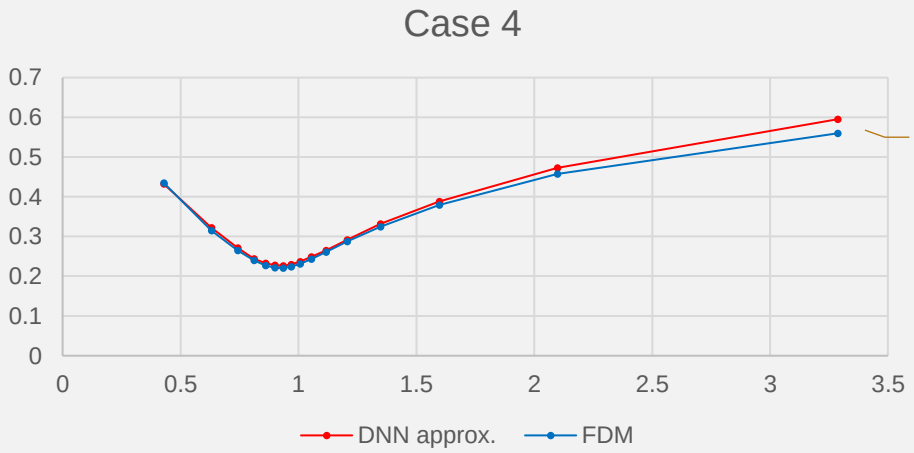
- Numerical result – drawing smile by DNN Approx. (2/2)







This strike point is near 5%-delta-put point. Then DNN approx. accuracy is not good because lower limit of training data is 5%-delta-put point and near there, training data density is low.



This difference is caused by both FDM error and DNN approx. error. FDM often cannot do well at far area on a smile wing.

PROTOTYPE TEST (9/19)

- Numerical result – Calibration and drawing smile curve (1/7)
  - Calibrating α_0, ρ, ν to real market volatility data and drawing smile curve
  - Comparing calibration results with other interpolation/extrapolation methods

Interpolation/extrapolation methods	Setting / optimization tool
Vanna volga	• Strike < 10% delta put - using 10% delta put/call volatility • 10%delta put ≤ strike < 25% delta put - using proportion of both 10% delta put volatility and 25% delta put volatility • 25%delta put ≤ strike ≤ 25% delta call - using 25% delta put/call volatility • 25%delta call < strike ≤ 10% delta put - using proportion of both 10% delta call volatility and 25% delta call volatility
Hagan' approximation	• Calibration: least_square() function in scipy • Interpolation/extrapolation: Hagan' approximation
DNN Approx.	• Calibration: least_square() function in scipy • Interpolation/extrapolation: DNN Approx.
DNN Calib.	• Calibration: keras model.predict_on_batch function • Interpolation/extrapolation: DNN Approx.

PROTOTYPE TEST (10/19)

- Numerical result – Calibration and drawing smile curve (2/7)
  - Calibrating real market data
    - Currency pair EUR/USD
    - Tenor 18M

Data date	Feb. 19, 2010		Mar. 13, 2013		Nov. 26, 2014		Sep. 17, 2018	
Forward rate	1.3366		1.3099		1.2544		1.2194	
Strike/IV	**strike**	**IV**	**strike**	**IV**	**strike**	**IV**	**strike**	**IV**
Extrapolation 1	1.002445		1.040789		1.003509		1.036479	
10-delta put	1.062461	0.157805	1.091882	0.118129	1.072153	0.105208	1.056535	0.095456
Interpolation 1	1.069274		1.170887		1.128947		1.097448	
25-delta put	1.20922	0.136976	1.204	0.102263	1.169026	0.091841	1.14335	0.083218
Interpolation 2	1.269763		1.235937		1.191667		1.158418	
ATM	1.352565	0.125513	1.309334	0.091422	1.261046	0.082993	1.224765	0.075346
Interpolation 3	1.470252		1.366035		1.317105		1.280356	
25-delta call	1.497181	0.122882	1.407418	0.086945	1.348958	0.080457	1.303164	0.073859
Interpolation 4	1.603912		1.431085		1.379825		1.341326	
10-delta call	1.654783	0.126944	1.504751	0.087059	1.440318	0.082585	1.38775	0.07725
Extrapolation 2	1.670741		1.561183		1.505263		1.463265	

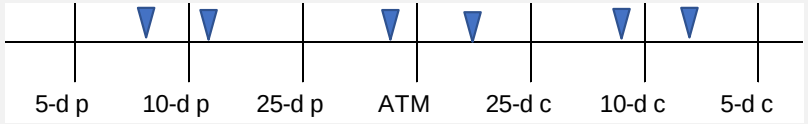
Volatilities are calculated using simple arithmetic, "market strangle."

$$\sigma^{C}_{delta} = \sigma_{ATM} + \frac{1}{2} RR_{delta} + BF_{delta}$$

$$\sigma^{P}_{delta} = \sigma_{ATM} - \frac{1}{2} RR_{delta} + BF_{delta}$$

  - Intepolation / extrapolation

Six strikes are picked arbitrarily as interpolation and extrapolation points from between each delta point, under "10-delta put" strike and over "10-delta call" strike.



PROTOTYPE TEST (11/19)

- Numerical result – Calibration and drawing smile curve (3/7)
  - Calibrated SABR parameters

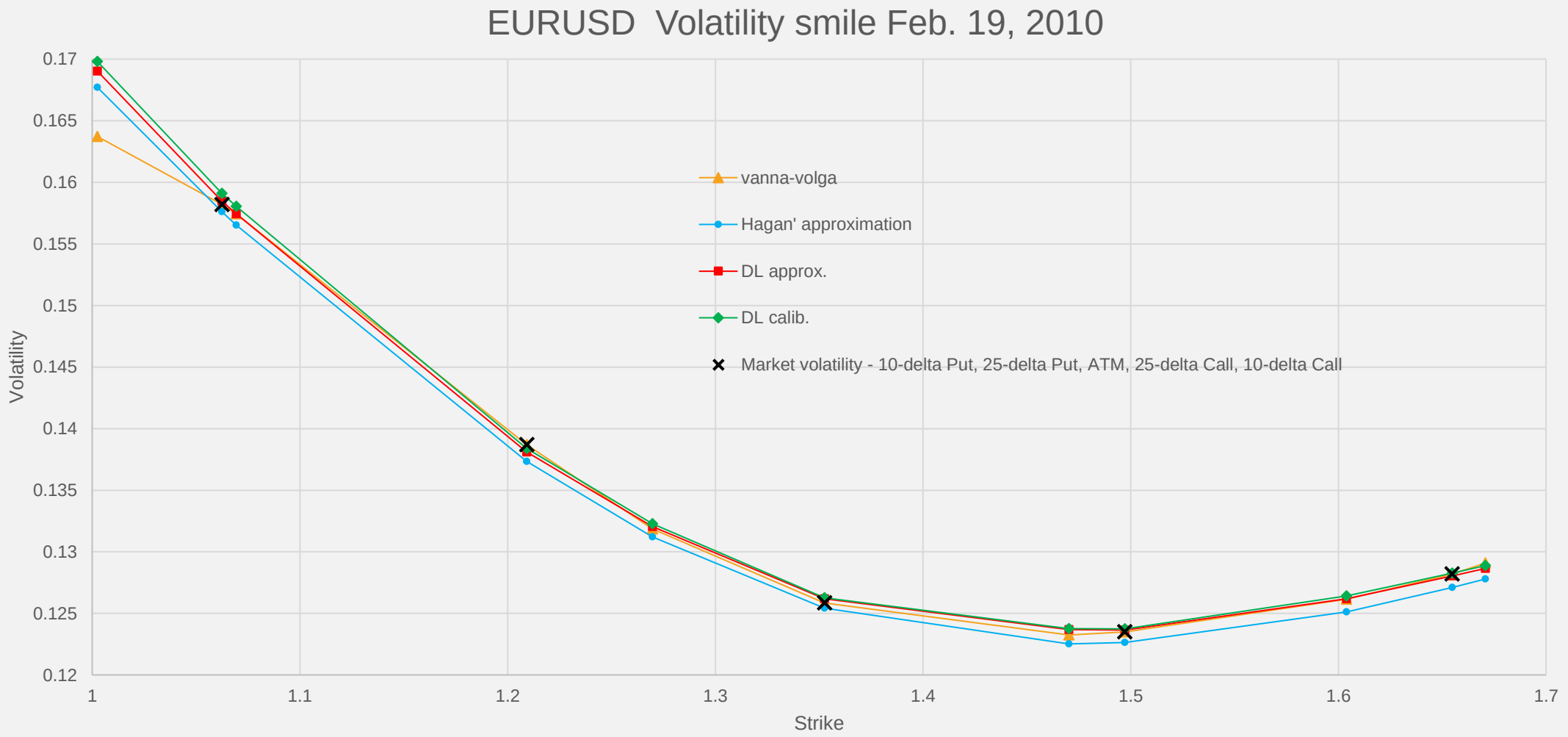
Data date	Feb. 19, 2010			Mar. 13, 2013			Nov. 26, 2014			Sep. 17, 2018		
	α	ρ	ν	α	ρ	ν	α	ρ	ν	α	ρ	ν
Hagan' approximation	0.124432	-0.17889	0.537617	0.07532101	-0.30901	0.506137	0.091739	-0.22419	0.524591	0.083126	-0.18343	0.553797
DNN Approx.	0.122935	-0.17374	0.556933	0.07415797	-0.31312	0.521621	0.090861	-0.23239	0.552005	0.081958	-0.18969	0.584813
DNN Calib.	0.12295	-0.17647	0.564586	0.074331	-0.31199	0.541931	0.091146	-0.22764	0.565128	0.082298	-0.18846	0.596481

  - Residual sum of squares at five market delta points

Data date	Feb. 19, 2010	Mar. 13, 2013	Nov. 26, 2014	Sep. 17, 2018	Remarks
Vanna-volga	3.11E-17	1.06E-17	1.07E-16	5.24E-17	Pass surely on the delta points
Hagan' approximation	4.31E-06	4.42E-06	5.19E-06	5.39E-06	Calibration hard
DNN Approx.	5.83E-07	2.85E-07	7.64E-07	3.25E-07	Pretty good?
DNN Calib.	1.14E-06	3.56E-06	2.89E-06	1.5E-06	Not bad?

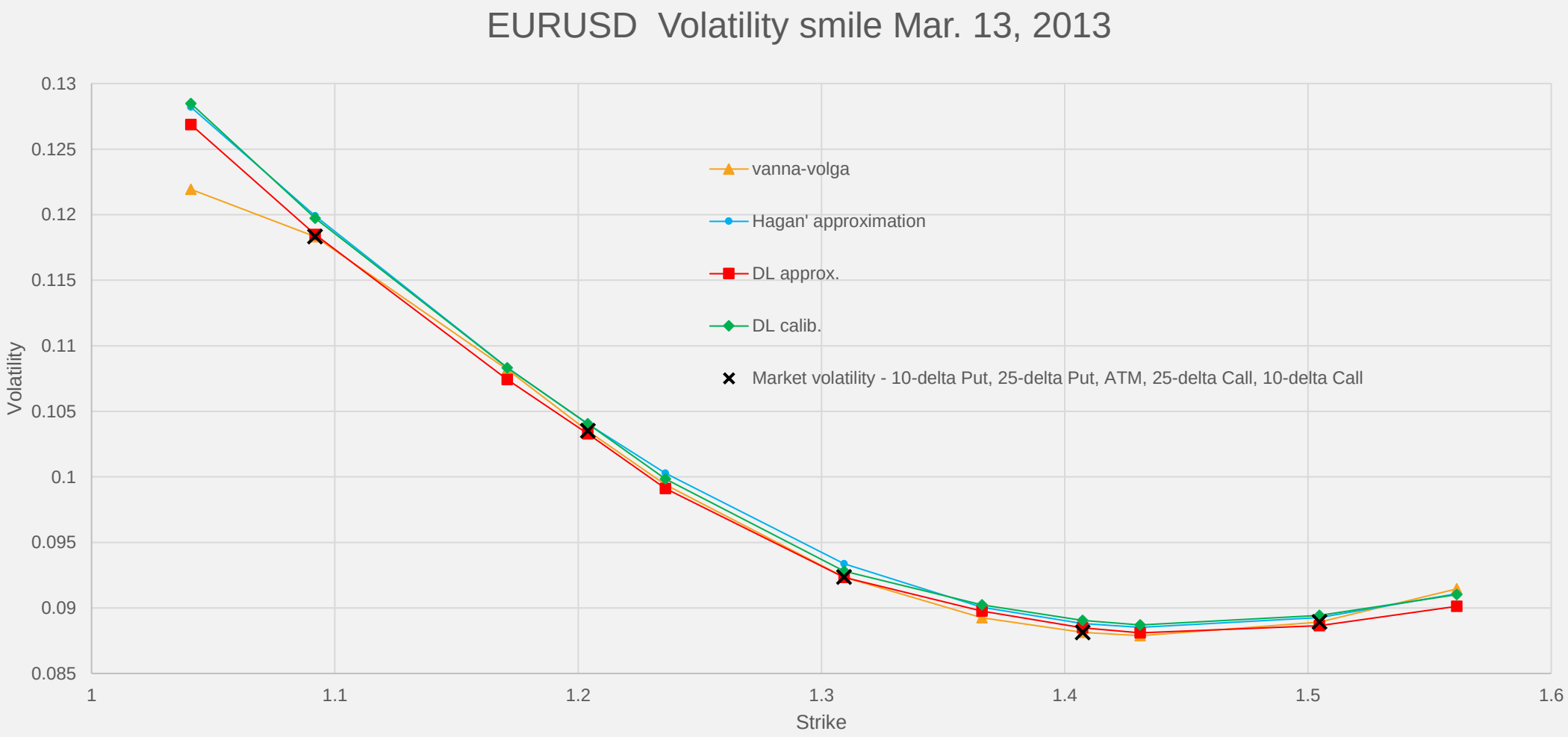
PROTOTYPE TEST (12/19)

- Numerical result – Calibration and drawing smile curve (4/7)



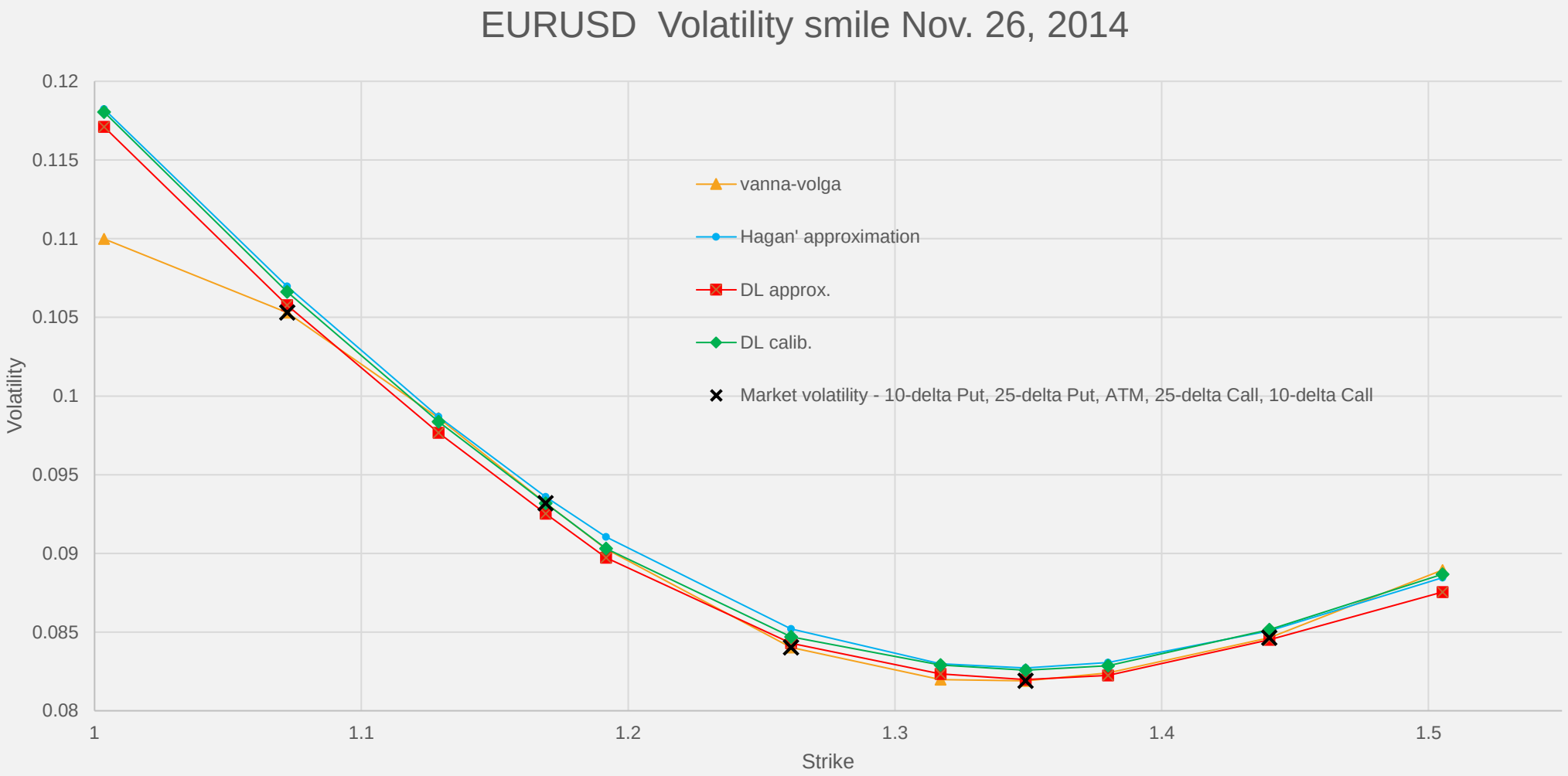
PROTOTYPE TEST (13/19)

- Numerical result – Calibration and drawing smile curve (5/7)



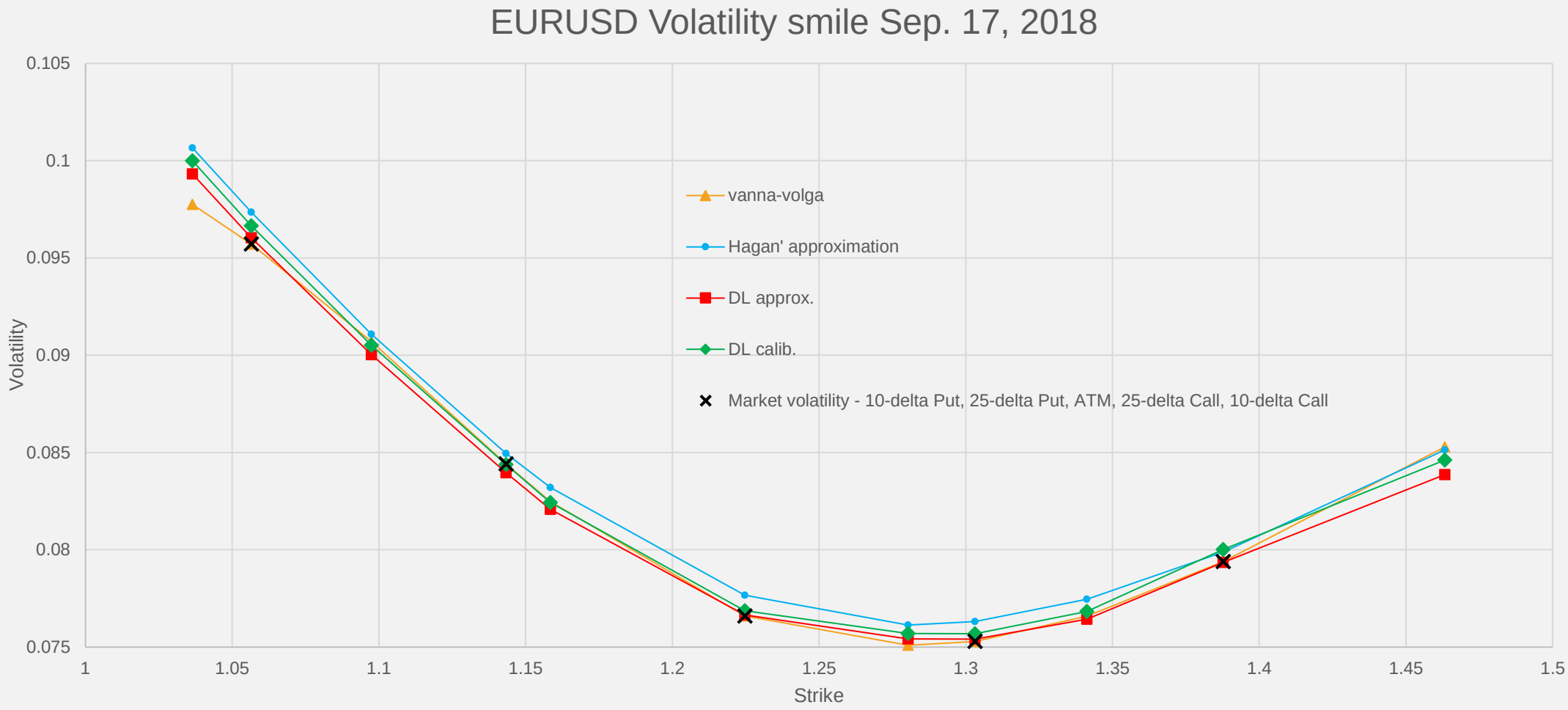
PROTOTYPE TEST (14/19)

- Numerical result – Calibration and drawing smile curve (6/7)



PROTOTYPE TEST (15/19)

- Numerical result – Calibration and drawing smile curve (7/7)



PROTOTYPE TEST (16/19)

- Performance (1/2)
  - Generating training data

	Value / number
MC condition	
Number of MC path	10,000,000
Number of time discretization	250/y
Output data	σ5dp, σ10dp, σ25dp, σATM, σ25dc, σ10dc, σ5dc, K5dp, K10dp, K25dp, KATM, K25dc, K10dc, K5dc, Arbitrary Strike, Volatility … 16 data
Elapsed time a MC simulation	3.25 seconds
Elapsed time for generating 1 million volatilities	3.25 x 1,000,000 / 8 ≈ 4.70 days

  - DNN learning[6]

	Time
DNN Approx. 1 epoch	1.34 seconds
DNN Calib. 1epoch	2.16 seconds

6 By using tf.data.Dataset, 5-10 times faster than before

PROTOTYPE TEST (17/19)

- Performance (2/2)
  - Calibration[7]

	Elapse Time
DNN Approx. using scypy' least_square() and tf.model.predit_on_batch()	About 1 seconds for a smile
DNN Calib.	About 0.1 seconds for a smile

7 it is known that tf.model.predict_on_batch is slow because it' language is Python.

PROTOTYPE TEST (18/19)

- Machine environment for prototyping
  - Generating Training data
    - OS Windows 11 Professional
    - Memory 32GB
    - CPU Core i7-10750H
    - GPU RTX2070 Super x 1
  - DNN
    - OS Ubuntu 18.04.6
    - Memory 64GB
    - CPU Xeon(R) CPU E5-2650 v2 x 2
    - GPU Tesla K20C x 2

PROTOTYPE TEST (19/19)

- Performance comparison between the prototyping GPUs and the latest GPUs

  The latest GPUs win overwhelming victories over the prototyping ones! The performance must be improved significantly if such high-speed GPUs are populated in the computers for generating test data and DNN learnings.

  As of the last day of Oct. 2022

	Prototyping	Latest	Prototyping	Latest
Model	RTX2070Super	RTX4090	Tesla K20C	Tesla H100
FP32	7.066 TFLOPS	73.073 TFLOPS	3.5 TFLOPS	66.9 TFLOPS
FP64	0.2 TFLOPS	1.142 TFLOPS	1.2 TFLOPS	33.5 TFLOPS
Remarks	• Sale date Jan. 2019 • Used for prototyping to generate training data • For consumer	• Sale date Sep. 2022 • For consumer	• Sale date Nov. 2012 • Used for prototyping to execute TensorFlow • For business	• Sale date Mar. 2022 • For business

APPLICATION EXAMPLE (1/2)

- Risk measure
  - Volatility surface transition as risk factor for FRTB-IMA
  - Sensitivity using numerical differentiation
- Accounting
  - Pricing options for both administrative and financial accounting
  - Pricing xVA for both administrative and financial accounting
- Front pricing
  - Alternative to vanna-volga interpolation
- R&D
  - To check existing SABR interpolation
  - For comparing another interpolation method

And so on.

APPLICATION EXAMPLE (2/2)

- <example> volatility surface transition as risk factor for FRTB-IMA
  a. Generate, for example, 250 volatility smiles on the risk horizon at every market convention tenor, such as 1D, 1W, …., 1Y, 2Y, 3Y, ….
  b. Calibrate SABR parameters to every smile using DNN Calib.
  c. Calculate volatilities on every option instrument in portfolios in every risk scenario using DNN Approx. with DNN Calib. result as initial value.

- Other usages of Universal Approximation Theorem outside SABR
  - Apply to compute probability on multivariate normal distribution for credit related measurement
  - Portfolio management
  - Calculating hedge amount

  And so on. It can cover a lot of ground.

CONCLUSION

- Numerical result are not bad, and performance is also not so bad albeit using poor hardware.
- Using DNN and GPU are probably super practical for financial institutions because the financial models which are hard to compute so far will be in practical use by means of them.
- This method is not a black-box because all values used for learning are theoretically made by oneself.
- And inferring from theoretical consideration,
  - Residual error is relatively big in some cases. Far strike from ATM, big ρ and big ν, and son. As a possible solution, expansion range of strike, increasing MC paths and increasing number of time-discretization are effective.
  - The issue above may be easily improved by using the latest hardware.
  - Furthermore, by increasing training data, DNN estimation accuracy and precision may be also improved.

TO CLOSE

- Thank you for reading this document.
- What do you think about this presentation? You think GPU and DNN have infinite possibilities, don’t you? If you think so, eigenView Inc. will give you support to your consideration of the introduction.
- If you would like to know more or have any questions, please email to the address below.
- And I can show you an actual example to compute volatility in an online meeting.
- Furthermore, if you provide me with some real market volatility values at that moment, I will calibrate the SABR parameters to them at once.

Best regards,

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eigenView Inc.

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